

AP CALCULUS/CALCULUS FINAL EXAM REVIEW 2018

FINAL: Friday, May 18, 2018 7:47-10:47 rooms 321 & 322

Show all work. Problems are due on the day of your final exam. Make sure you know basic graphs, the unit circle, special triangles, trig identities, rules for derivatives (including the limit definition) and integration, Newton's Method, L'Hopital's Rule, Fundamental Theorem of Calc, applications of the derivative or integral problems... **NO CALCULATOR!!!**

PreCalculus:

Simplify each:

1. $e^{\ln x}$

4. $\ln(e^{(5x)})$

2. $e^{8\ln(x)}$

5. $\sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

3. $2\ln(3x) - \ln(5x^4)$

6. $\sin\left(\cos^{-1}\left(\frac{1}{4}\right)\right)$

7. A culture grows with a constant growth rate. Use the formula $y = y_0 a^t$ to find the expression for the culture if after 1 hour there were 25 bacteria and after 4 hours there were 200 bacteria.

8. Find the inverse of the function $y = \frac{3x-5}{x-1}$

Find each limit.

$$9. \lim_{x \rightarrow 2^+} \frac{1}{2-x}$$

$$17. \lim_{x \rightarrow 1} \frac{\ln x}{x^2-1}$$

$$10. \lim_{x \rightarrow 2^-} \frac{|(x-2)(x+5)|}{x-2}$$

$$18. \lim_{x \rightarrow \infty} \frac{x^4}{e^{2x}}$$

$$11. \lim_{x \rightarrow 0^+} \ln x$$

$$12. \lim_{x \rightarrow \infty} \ln x$$

$$19. \lim_{x \rightarrow \infty} \frac{e^{2x}}{x^4}$$

$$13. \lim_{x \rightarrow \infty} (\ln x - \ln(5x + 1))$$

$$14. \lim_{x \rightarrow 2} (3x^4 + 2x - 5)$$

$$20. \lim_{x \rightarrow -\infty} \frac{x^3}{x+1}$$

$$15. \lim_{x \rightarrow 0} (3x)^x$$

$$21. \lim_{x \rightarrow -\infty} \frac{3x^4 - 4x + 2}{5x^2 - 2x + 1}$$

$$16. \lim_{x \rightarrow \infty} (1 - e^{-x})^{1/x}$$

$$22. \lim_{x \rightarrow 0} \frac{\tan(2x)}{\ln(1-x)}$$

Find each derivative. (Do not simplify)

23. $y = \sin^2(4x^3)$

30. $y = \frac{e^{5x} \cos(6x)}{6x^3}$

24. $y = 5^{(3x+1)}$

31. $y = \tan(-4x^3) + \sec(4x + 1)$

25. $y = \frac{e^{5x}}{2x+3}$

32. $y = \sin^{-1}(4x)$

26. $y = 3x(x - 2)^3(3x - 4)$

33. $y = \tan^{-1}(-4x^3)$

27. $y = e^{-x} + \ln(e^{-2x}) + \frac{1}{2e^{-x}}$

34. $y = x^{\sin(4x)}$

28. $y = \ln(5x^6)$

35. $y = \ln\left(\frac{4x}{x^2 - 3}\right)^2$

29. $y = (\ln(5x))^6$

36. $y = \int_0^{4x} \sin(t^3) dt$

Integrate each.

$$37. \int_0^{\frac{\pi}{2}} \cos(3x) dx$$

$$42. \int \frac{\ln x}{x} dx$$

$$38. 6 \int (7x-2)^8 dx$$

$$43. \int_1^2 \frac{1}{x^4} dx$$

$$39. \int x \cos(3x^2) dx$$

$$44. \int \frac{x}{\sqrt{1-x^2}} dx$$

$$40. \int e^{6x} dx$$

$$45. \int \frac{5}{\sqrt{1-9x^2}} dx$$

$$41. \int \sec(4x) \tan(4x) dx$$

$$46. \int \frac{5}{1+9x^2} dx$$

$$51. \int \tan^{-1} x dx$$

$$47. \int x \sin x dx$$

$$52. \int_1^2 (2x - 1) dx$$

$$48. \int 5 \ln x dx$$

$$53. \int \frac{2x-4}{x^2-4x+3} dx$$

$$49. \int \cos^2(5x) dx$$

$$54. \int \frac{6x-12}{x^2-4x+3} dx$$

$$50. \int \sin^3(5x) dx$$

55. Given $3x^2 + 4x^2y + xy = 8$. Find the equation of the tangent line at the point (1,1).

56. Given $(x + y)^2 - 2y^2 + x = 2$. Find the linearization (tangent line) at the point (1,2). Then, use the linearization to approximate y when $x = 1.1$.

57. Given : $x(t) = 3t$ and $y(t) = 9t^2$, find the equation of the tangent line when $t=1$.

58. Given : $x(t) = \sin t - 4t$ and $y(t) = \cos(2t) + 6t$, find the equation of the tangent line when $t = \pi/2$.

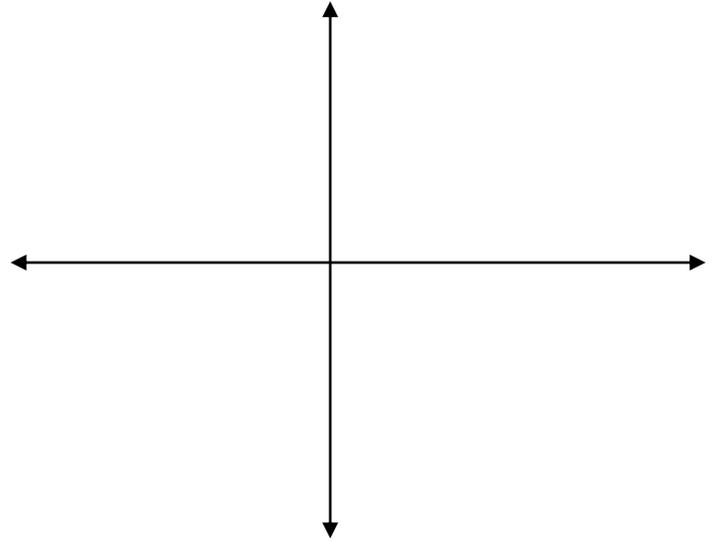
59. Given the formula & (b, a) , $(f^{-1}(a))' = \frac{1}{f'(b)}$. Find $(f^{-1}(-3))'$ given $f(x) = 4x^3 + x^2$ & $f(-1)=-3$

60. Given the formula (b, a) , $(f^{-1}(a))' = \frac{1}{f'(b)}$. Find $(f^{-1}(2))'$ given $f(x) = \frac{x-5}{x-3}$ & $f(1)=2$

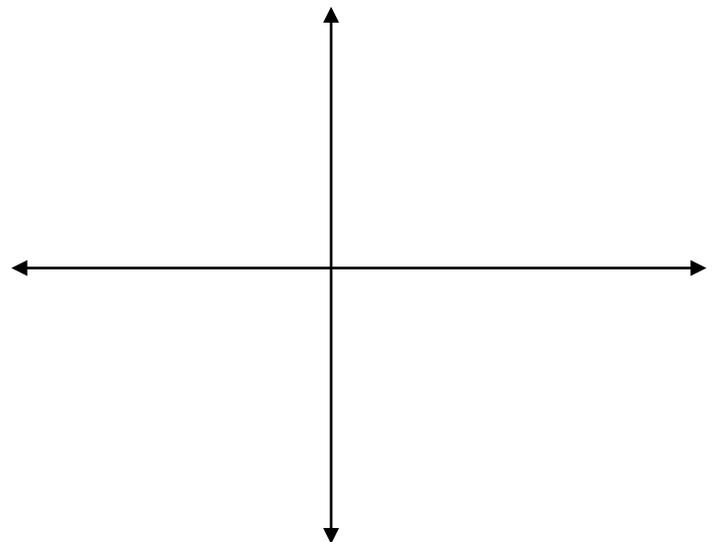
61. Given a rectangular box with a square base and open top and a volume of 50 cubic feet. If the cost for the material for the bottom of the box is $\$2/\text{ft}^2$ and for the sides is $\$1.5/\text{ft}^2$, find the dimensions of the box which will minimize the cost.
62. A farmer wants to enclose a rectangular area (one side is along a river) with 100 feet of fence. What should the dimensions be to maximize the area?
63. A rectangle is inscribed under a parabola with the vertex at the point $(0,9)$ and passing through the point $(3,0)$. Find the length and width of the rectangle with the largest area.

64. A 13 foot ladder is leaning against a wall when it's base starts to slide away. When the base is 12 feet from the wall, the base is moving at a rate of 5 ft/sec. How fast is the top of the ladder sliding down the wall at this time?
65. Fred flies a kite at a height of 300 feet. The wind carries the kite horizontally away at a rate of 25 ft/sec. How fast must she let out the string when it is 500 feet away? How fast is the angle made with the kite string and the ground changing at this time?
66. If the area of a circle is increasing at a rate of 100 square feet per min, how fast is the circumference changing when the radius is 10 feet?

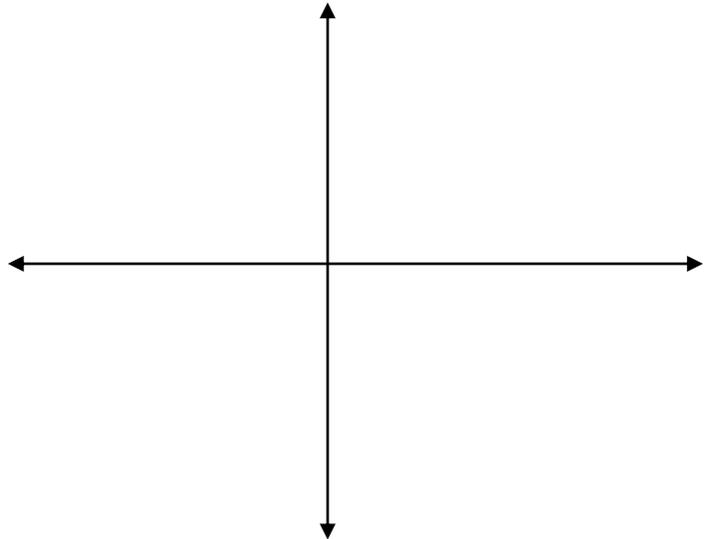
67. Given the function $y = -x^3 + 4.5x^2 - 6x - 1$. Draw a complete graph including asymptotes, local max & min points, & intervals of inc/dec & concavity.



68. Given the function $y = \frac{e^x}{x}$. Draw a complete graph including asymptotes, local max & min points, & intervals of inc/dec & concavity.



69. Given $f(x) = \frac{x}{x^2-4}$, $f'(x) = \frac{-(x^2+4)}{(x^2-4)^2}$ & $f''(x) = \frac{2x(x^2+12)}{(x^2-4)^3}$ find any vertical and horizontal asymptotes, intervals of inc/dec and concavity. Draw a sketch indicating critical points, inflection points and asymptotes.

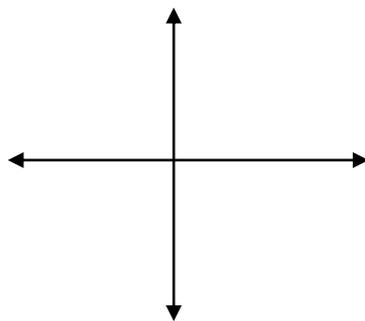
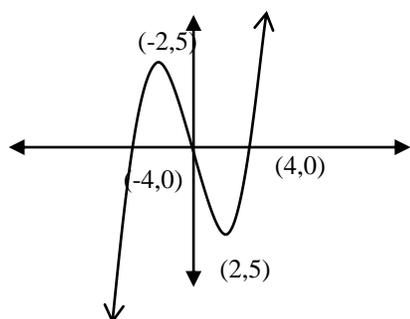


70. Using Newton's method, find an approximate value of $\sqrt{82}$. Determine an equation, find the initial guess and run through one iteration of Newton's method by hand.

71. Given the function $y = \frac{4x}{x^2+2}$, find the absolute max and min values (range).

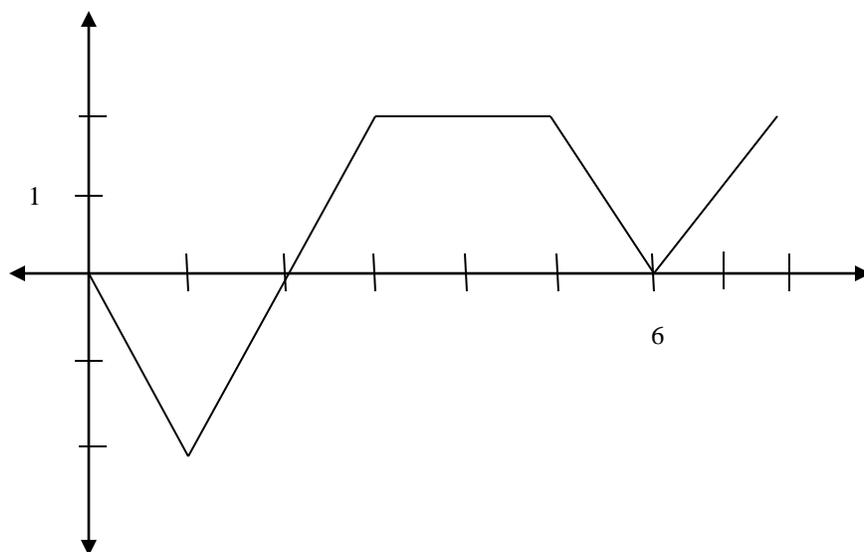
72. Given the velocity equation $v = t^2 - 3t + 2$ for a function, find the displacement of the object on the time interval $[0,2]$ and the total distance of the object on the time interval $[0,2]$.

73. Given the following function. Draw a sketch of the derivative.



74. Given the following derivative function $f'(x)$. Use geometry to find the integrals listed below. Fill in the table of values and sketch the function $f(x)$. Be sure to show local max/min and inflection points.

x	y
0	0
1	
2	
3	
5	
6	
8	



$$\int_0^2 v(t) dt =$$

$$\int_2^6 v(t) dt =$$

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